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FREE EBOOK

SCIENCE PROJECTS AND KITS

Maths Working Models and Activities

Make maths visible: models for number, geometry, algebra and data

INSIDE THIS BOOK

- Making maths visible
- Place value
- The number line and integers
- Even, odd and prime numbers
- ...and 7 more chapters, practice questions and a glossary

Beginner to intermediate

Students of class 3 to 10 and maths teachers

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Maths Working Models and Activities

Hands-on maths models you can make from card, paper, string and bottle caps, covering place value, integers, fractions, primes, tables, angles, Pythagoras theorem, area, volume, algebraic identities, probability, bar graphs and a clinometer for measuring heights. Each model comes with clear steps, worked examples and a simple way to explain the idea to a teacher or judge.

WHO IT IS FOR	Students of class 3 to 10 and maths teachers
LEVEL	Beginner to intermediate
SUBJECT AREA	Science projects and kits
WHAT IS INSIDE	11 chapters, practice questions with answers, and a glossary

HOW TO USE IT

Read one chapter at a time and try the examples as you go: on a computer, at your workbench or in your notebook. The boxes give tips, safety points and short activities. When you finish, test yourself with the practice questions and check your answers at the end. Words you may not know are explained in the glossary.



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Making maths visible

Many students find maths hard because it lives only in symbols on a page. A model brings it into your hands. When you fold, cut, slide and rearrange real pieces, you can see why a rule is true instead of just remembering it.

A **working model** in maths is one that does something: its pieces move, can be rearranged, or can be measured to test a rule. A poster of a formula is not a working model. Four squares of card that you can rearrange to prove the formula are.

Materials

Material	Used for
Chart paper and old cartons	Bases, strips, shapes
Graph paper and dot paper	Accurate squares and grids
Geometry box	Measuring and drawing angles and circles
Scissors, glue and cello tape	Cutting and joining
Split pins (paper fasteners)	Spinners and wheels that turn
Bottle caps, beads, bindis	Counters and markers
Thread, string and a small weight	Measuring curves; the clinometer

SAFETY

An adult should handle sharp cutters, hammering and hot glue. Point the compass needle away from your body and close it after use.

Measure carefully

A maths model must be accurate, because it is proving something. If your squares are 2 mm out, a judge can say your proof only looks right. Use a sharp pencil, measure twice and cut exactly on the line.

How to explain any model

Use these five steps for every model in this book:

1. **Name the idea.** "This model shows the angle sum of a triangle."
2. **Show it working.** Move or rearrange the pieces.
3. **Let the viewer try.** Hand them a piece. People remember what they do.
4. **Connect it to the rule.** "So the three angles always make 180° ."
5. **Give a real use.** Where does this idea help in daily life?

EXAMPLE

Ananya from Indore made four models for her school maths fair. She bought chart paper (! 40), graph paper (! 20), a packet of split pins (! 30) and thread (! 10). Bottle caps and cartons came from home. Her total was ! 100.

REMEMBER

Judges reward understanding more than decoration. A plain model you can explain well beats a shiny one you cannot.

CHAPTER 2

Place value

The idea

In the number 555, every digit is a 5, yet each one is worth something different: 500, 50 and 5. The value of a digit depends on its **place**. This is **place value**. The digit itself, without its place, is its **face value**. In 3,527, the face value of 5 is 5, but its place value is 500.

Each place is worth ten times the place to its right. That is why we need only ten digits, 0 to 9, to write any number.

Model 1: the straw pocket chart

You need: three paper pockets glued in a row on a chart, labelled H (hundreds), T (tens) and O (ones), a bundle of straws and rubber bands.

1. Count straws into the ones pocket.
2. When you reach ten, tie them with a rubber band into one bundle and move it to the tens pocket.
3. When you have ten bundles, band them together into a hundred and move it to the hundreds pocket.
4. To show 243, put 2 big bundles, 4 bundles and 3 single straws in the pockets.

Model 2: the spike abacus

You need: a thermocol block, seven wooden skewers and plenty of beads.

1. Push the skewers into the block in a row.
2. Label them from right to left: O, T, H, Th (thousands), TTh (ten thousands), L (lakhs) and TL (ten lakhs).
3. Show a number by putting beads on each skewer. To show 3,52,407, put 3 beads on L, 5 on TTh, 2 on Th, 4 on H, none on T and 7 on O.

A skewer never holds more than nine beads. When a tenth bead goes on, take all ten off and put one bead on the next skewer to the left. This is **carrying**, made visible.

Indian and international systems

Number	Indian system	International system
3,52,407	Three lakh fifty-two thousand four hundred seven	352,407: three hundred fifty-two thousand four hundred seven
1,00,00,000	One crore	10,000,000: ten million

In the Indian system, the first comma comes after three digits from the right, and then after every two. In the international system, a comma comes after every three digits.

EXAMPLE

Rahul paid ₹ 347 for a geometry box with three ₹ 100 notes, four ₹ 10 notes and seven ₹ 1 coins. The notes and coins are place value in action: hundreds, tens and ones.

TIP

Ask the viewer what an empty skewer means. The answer, zero, is the key point: zero holds a place, so 407 is not the same as 47.

CHAPTER 3

The number line and integers

The idea

Integers are the whole numbers and their negatives: ... -3, -2, -1, 0, 1, 2, 3 ... On a **number line**, positive numbers lie to the right of zero and negative numbers to the left. Every number is larger than any number to its left, so -2 is larger than -5.

Negative numbers describe real things: a temperature below zero, a basement floor, money owed, or a place below sea level.

Model 1: the walking number line

You need: a long strip of tape or chart paper on the floor, marked from -10 to +10 with steps about 30 cm apart.

The rules:

1. Start on the first number.
2. For **plus**, face the positive end. For **minus**, face the negative end.
3. If the second number is positive, walk **forwards**. If it is negative, walk **backwards**.

Try these:

- $2 - 5$: start at 2, face the negative end, walk forwards 5 steps. You land on -3 .
- $-4 + 6$: start at -4 , face the positive end, walk forwards 6. You land on 2.
- $-4 - (-6)$: start at -4 , face the negative end, walk backwards 6. You land on 2.

The last one shows why subtracting a negative number is the same as adding a positive one.

Model 2: zero-pair counters

You need: bottle caps in two colours. Let green stand for $+1$ and red for -1 .

A green and a red together make zero. This is a **zero pair**.

- $-3 + 5$: put down 3 red and 5 green. Remove 3 zero pairs. Two green are left, so the answer is $+2$.
- $2 - 5$: put down 2 green. You need to take away 5 green but have only 2, so add 3 zero pairs. That does not change the value. Now take away 5 green. Three red are left, so the answer is -3 .

EXAMPLE

At 6 a.m. on a winter morning in Leh, the temperature was -8°C . By noon it had risen by 12 degrees. On the number line, start at -8 and walk 12 steps towards the positive end: the noon temperature was 4°C .

TRY THIS

In a shopping mall, the ground floor is 0 and the parking levels are -1 and -2 . If you are on -2 and go up 5 floors, which floor are you on? (Answer: 3.)

CHAPTER 4

Even, odd and prime numbers

Even and odd

You need: 20 bottle caps.

Take any number of caps and arrange them in pairs. If every cap has a partner, the number is **even**. If one cap is left over, the number is **odd**. Zero is even, because there are no caps left over.

Now put two groups together and see what happens:

Add	Result	Why
Even + even	Even	All caps are already paired
Odd + odd	Even	The two leftover caps pair up
Even + odd	Odd	One cap is still left over

The sieve of Eratosthenes

A **prime number** has exactly two factors: 1 and itself. 2, 3, 5 and 7 are prime. A number with more than two factors, such as 6 (1, 2, 3 and 6), is **composite**. The number 1 has only one factor, so it is neither prime nor composite.

The Greek mathematician Eratosthenes, who lived more than 2,000 years ago, found primes with a method like a sieve: you remove the numbers you do not want and keep the rest.

You need: a 10 by 10 grid of the numbers 1 to 100 on chart paper, and bindis in four colours.

1. Cross out 1.
2. Leave 2 itself bare, and put a red bindi on every multiple of 2 after it: 4, 6, 8 and so on.
3. Leave 3 bare, and put a blue bindi on every multiple of 3 after it: 6, 9, 12 and so on.
4. Do the same for 5 with green and for 7 with yellow.
5. Every number with no bindi, apart from 1, is prime. Circle them. You will find 25 primes.

You can stop after 7 because $11 \times 11 = 121$, which is larger than 100. Any composite number up to 100 has a factor of 7 or less, so it already has a bindi.

DID YOU KNOW

Your sieve also shows factors. The number 30 carries red, blue and green bindis, because 2, 3 and 5 all divide it. The multiples of 2 and 5 fall in straight columns, while the multiples of 3 run in slanting lines.

The multiplication wheel

You need: two circles of card, 20 cm across, and a split pin.

1. Divide the bottom circle into ten equal sectors (36° each), like slices.
2. In each sector, write a question near the centre ($7 \times 1, 7 \times 2 \dots 7 \times 10$) and its answer near the edge (7, 14 ... 70).
3. In the top circle, cut two windows in one sector: a small one near the centre and another near the edge. Tape a paper flap over the outer window.
4. Join the circles at their centres with the split pin.
5. Turn the top circle to show a question, say the answer, then lift the flap to check.

Make a bottom circle for each table from 2 to 10. One top circle serves them all.

TRY THIS

Mark the digits 0 to 9 round a circle like a clock face. Join the last digits of the 4 times table in order: 4, 8, 2, 6, 0. You draw a five-pointed star. The 2 times table (2, 4, 6, 8, 0) makes a pentagon, and the 3 times table touches all ten points.

CHAPTER 5

Fractions

The idea

A **fraction** describes part of a whole that has been divided into **equal** parts. In $\frac{3}{4}$, the bottom number, the **denominator**, says the whole is cut into 4 equal parts. The top number, the **numerator**, says we take 3 of them.

Model 1: the fraction wall

You need: coloured chart paper, a ruler and a large envelope.

1. Cut seven strips, each 24 cm long and 3 cm wide, in seven colours.
2. Leave the first strip whole. This is 1.
3. Cut the other strips into equal pieces: halves (12 cm each), thirds (8 cm), quarters (6 cm), sixths (4 cm), eighths (3 cm) and twelfths (2 cm).
4. Write the fraction on every piece.
5. Lay the strips in rows under the whole, like the bricks of a wall. Keep the pieces loose so they can be moved.

The length 24 cm is chosen because 24 can be divided exactly by 2, 3, 4, 6, 8 and 12.

What the wall shows

- **Equivalent fractions:** one half-piece is the same length as two quarters, three sixths, four eighths or six twelfths. So $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{6}{12}$.
- **Comparing:** two thirds measure 16 cm and three quarters measure 18 cm, so $\frac{3}{4}$ is larger than $\frac{2}{3}$.
- **Adding:** lay a half (12 cm) and a third (8 cm) end to end. Together they measure 20 cm. Which row matches 20 cm? Five sixths (5×4 cm). So $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$.

Model 2: fraction circles

Draw circles of the same size and divide them into equal sectors with a protractor. The angle of each sector is 360° divided by the denominator: 180° for halves, 120° for thirds, 90° for quarters, 60° for sixths and 45° for eighths.

REMEMBER

The parts must be equal. A roti torn into three uneven pieces has not been divided into thirds.

EXAMPLE

Priya had ₹ 240 and spent $\frac{3}{8}$ of it on books. First find one eighth: ₹ $240 \div 8 = ₹ 30$. Then three eighths: $3 \times ₹ 30 = ₹ 90$. On the fraction wall, three eighth-pieces cover the same length as $\frac{3}{8}$ of the whole strip.

Angles, triangles and tessellations

The angle sum of a triangle

The idea: The three angles inside any triangle add up to 180° .

You need: coloured paper, a ruler, scissors and a sheet of card.

1. Draw any triangle and cut it out.
2. Colour its three corners red, blue and green on the front.
3. Tear off the three corners. Tearing leaves a rough edge, so you can always tell the torn edge from the triangle's straight sides.
4. On the card, draw a straight line and mark a point on it.
5. Place the three corners with their points meeting at that point, side by side.

They fit exactly along the straight line. A straight line makes an angle of 180° , so the three angles add up to 180° . Repeat with a thin, sharp triangle and with a triangle that has one wide angle. It works every time.

Exterior angle

Extend one side of a paper triangle with a ruler line. The angle formed outside is the **exterior angle**. Tear off the two corners that are not next to it and place them inside it. They fill it exactly: an exterior angle equals the sum of the two opposite interior angles.

Any polygon

The four corners of any quadrilateral, torn off and placed together, fill a full turn of 360° . A quadrilateral can be split into two triangles, so $2 \times 180^\circ = 360^\circ$. A shape with n sides splits into $(n - 2)$ triangles, so its angles add up to $(n - 2) \times 180^\circ$.

Tessellations

A **tessellation** is a pattern of shapes that covers a flat surface with no gaps and no overlaps, like floor tiles. Wherever corners meet, their angles must add up to 360° .

Regular shape	Each angle	Meeting at a point	Tessellates?
Equilateral triangle	60°	$6 \times 60^\circ = 360^\circ$	Yes
Square	90°	$4 \times 90^\circ = 360^\circ$	Yes
Regular pentagon	108°	$3 \times 108^\circ = 324^\circ$, leaving a gap	No
Regular hexagon	120°	$3 \times 120^\circ = 360^\circ$	Yes

Only these three regular shapes, the triangle, the square and the hexagon, tessellate on their own. But any triangle tessellates, and so does any quadrilateral, because their angles can always be arranged to make 360° at each point.

Make a tessellation: cut 20 identical shapes from card and fit them together on a board. For a more artistic version, cut a curvy piece out of one side of a card square and tape it exactly onto the opposite side. The new shape still tessellates. Trace round it again and again, then colour the pattern.

DID YOU KNOW

Honeybees build their combs from hexagons. Hexagons tessellate, and of the three regular shapes that do, they enclose the most space for a given length of wall.

CHAPTER 7

Pythagoras theorem

The idea

In a right-angled triangle, the longest side, opposite the right angle, is the **hypotenuse**. **Pythagoras theorem** says that the square on the hypotenuse equals the sum of the squares on the other two sides:

$$a^2 + b^2 = c^2$$

where c is the hypotenuse and a and b are the other two sides.

Model 1: squares on the sides

You need: 1 cm graph paper, coloured card, glue and scissors.

1. On graph paper, draw a right-angled triangle with sides of 3 cm and 4 cm around the right angle. Join the ends. The hypotenuse measures 5 cm.
2. Draw a square on each side: 3 by 3, 4 by 4 and 5 by 5.
3. Count the small squares: 9, 16 and 25.
4. Cut the 9 square and the 16 square into single small squares. Fit all 25 of them onto the big square. They cover it exactly.

So $9 + 16 = 25$, or $3^2 + 4^2 = 5^2$.

TIP

Use 2 cm squares instead (sides of 6, 8 and 10 cm) for a model that people can see from a distance.

Model 2: the four-triangle proof

This model proves the theorem for every right-angled triangle, not just one.

You need: card, a ruler and scissors.

1. Cut four identical right-angled triangles with sides of 6 cm and 8 cm around the right angle. The hypotenuse is 10 cm.
2. Make a square frame with inside sides of 14 cm (that is, $6 + 8$).
3. **Arrangement 1:** place a triangle in each corner of the frame, with its right angle in the corner. Turn them so that every side of the frame has one 6 cm side and one 8 cm side along it. The empty space in the middle is a tilted square whose sides are the hypotenuses. Its area is c^2 .
4. **Arrangement 2:** rearrange the four triangles into two rectangles, each 6 cm by 8 cm, in opposite corners of the frame. Now the empty space is two squares: one 6 by 6 (a^2) and one 8 by 8 (b^2).

The frame is the same and the four triangles are the same, so the empty space must be the same too. Therefore $c^2 = a^2 + b^2$.

a	b	c	Check
3	4	5	$9 + 16 = 25$
6	8	10	$36 + 64 = 100$
5	12	13	$25 + 144 = 169$
8	15	17	$64 + 225 = 289$

Real uses

Masons check that the corner of a room is a right angle by measuring 3 units along one wall and 4 along the other. If the distance between the two marks is exactly 5, the corner is square. A ladder 5 m long, with its foot 3 m from a wall, reaches 4 m up the wall.

DID YOU KNOW

An ancient Indian text on building fire altars, the Baudhayana Sulba Sutra, states this rule for the diagonal of a rectangle.

CHAPTER 8

Area, perimeter and the circle

The idea

Perimeter is the distance all the way round a shape. **Area** is the amount of flat space inside it, measured in square units such as square centimetres (cm^2).

Model 1: the string rectangle

You need: a thermocol sheet covered with 1 cm graph paper, four pins and a loop of string exactly 24 cm long.

1. Put the loop round the four pins and stretch it into a rectangle along the grid lines.
2. Count the squares inside to find the area.
3. Move the pins to make a different rectangle, still using the whole loop.

Length	Width	Perimeter	Area
11 cm	1 cm	24 cm	11 cm ²
9 cm	3 cm	24 cm	27 cm ²
7 cm	5 cm	24 cm	35 cm ²
6 cm	6 cm	24 cm	36 cm ²

The perimeter stays the same, but the area changes. The square gives the largest area.

EXAMPLE

Mr Hegde in Mysuru has 36 m of fencing for a rectangular vegetable garden. A 9 m by 9 m square gives the largest garden, 81 m². If fencing costs ₹ 150 a metre, the fence costs $36 \times ₹ 150 = ₹ 5,400$ whichever shape he chooses, so the square gets him the most garden for the same money.

Model 2: the area of a circle

1. Draw a circle with a radius of 7 cm on thick paper and cut it out. The **radius** is the distance from the centre to the edge.
2. Fold it in half four times, then open it. The creases divide it into 16 equal sectors.
3. Colour one half of the circle yellow and the other half blue, then cut out the sectors.
4. Lay the yellow sectors in a row, points up. Fit the blue sectors between them, points down.

The pieces make a shape very close to a rectangle. Its height is the radius, r . Its length is half the distance round the circle. The distance round a circle, its **circumference**, is $2\pi r$, so half of it is πr . The area of the rectangle, and so of the circle, is $\pi r \times r = \pi r^2$.

For our circle: area = $22/7 \times 7 \times 7 = 154 \text{ cm}^2$.

TRY THIS

Wrap a thread round a steel plate and measure it. Then measure the plate's diameter, the distance across through the centre. For a plate 21 cm across, the thread measures about 66 cm, and $66 \div 21$ is about 3.14. Every circle gives the same answer. That number is π (pi).

DID YOU KNOW

The Indian mathematician Aryabhata, writing in 499 CE, gave a value of π equal to 3.1416.

CHAPTER 9

Volume of solids

The idea

Volume is the amount of space a solid takes up. It is measured in cubic units, such as cubic centimetres (cm^3). A **cube** with sides of 1 cm has a volume of 1 cm^3 . One litre is $1,000 \text{ cm}^3$.

Model 1: building a cuboid

You need: 30 small cubes of the same size, such as dice, sugar cubes or cubes folded from card.

1. Lay a single layer 4 cubes long and 3 cubes wide. It uses 12 cubes. This is the area of the base.
2. Add a second layer on top. Now there are 24 cubes.
3. The **cuboid** is 4 long, 3 wide and 2 high, and its volume is $4 \times 3 \times 2 = 24$ cubic units.

So the volume of a cuboid = length $\times$ breadth $\times$ height. It is also base area $\times$ height, which is the idea behind every formula in this chapter. A cube has all sides equal, so its volume is side $\times$ side $\times$ side, written a^3 .

Model 2: the litre box

1. Draw the **net** of a cube on card: six squares of 10 cm in a cross shape, with flaps for gluing. A net is the flat shape that folds into a solid.
2. Cut it out, fold it and glue it into a 10 cm cube, leaving the top open.
3. Line it with a thin plastic bag, stand it in a tray and pour in a one-litre packet of water.

The water fills the box to the brim, because $10 \times 10 \times 10 = 1,000 \text{ cm}^3$, which is one litre.

Model 3: cylinders

Stack twenty ! 1 coins. Each coin is a thin circle, and the stack is a **cylinder**. Its volume is the area of the circle at the base multiplied by the height:

$$\text{volume of a cylinder} = \pi r^2 h$$

EXAMPLE

A tin has a radius of 7 cm and a height of 10 cm. Its volume = $\frac{22}{7} \times 7 \times 7 \times 10 = 1,540 \text{ cm}^3$, which is just over one and a half litres.

TRY THIS

Take two identical A4 sheets. Roll one into a tall, thin tube and the other into a short, wide tube, and tape them. Stand both on a tray and fill them with rice. Which holds more? Most people guess they are the same, but the short, wide one holds about $1,470 \text{ cm}^3$ and the tall one about $1,040 \text{ cm}^3$. The radius is squared in the formula, so a wider tube gains more than a taller one.

Algebraic identities

The idea

An **identity** is an equation that is true for every value of its letters. The identity $(a + b)^2 = a^2 + 2ab + b^2$ works whether a and b are 1 and 2 or 70 and 30. Area models show why.

Model 1: $(a + b)^2$

You need: card in three colours, a ruler and scissors. Take $a = 7$ cm and $b = 3$ cm.

1. Cut one square 7 cm by 7 cm in red. Its area is a^2 .
2. Cut one square 3 cm by 3 cm in blue. Its area is b^2 .
3. Cut two rectangles 7 cm by 3 cm in yellow. Each has area ab .
4. Fit the four pieces together into one large square.

The large square has sides of $7 + 3 = 10$ cm, so its area is $(a + b)^2$. It is made of a^2 , two ab and b^2 . Therefore $(a + b)^2 = a^2 + 2ab + b^2$.

Check with numbers: $10^2 = 100$, and $49 + 21 + 21 + 9 = 100$.

Model 2: $(a - b)^2$

Lay two yellow 7 by 3 strips over the red 7 by 7 square: one along the right side and one along the top. The part of the red square still showing is a square with sides of $7 - 3 = 4$ cm. The two strips cover $2 \times 21 = 42$ cm², but they overlap in a 3 by 3 corner, so that corner has been counted twice. The area still showing is therefore $49 - 42 + 9$. So $(a - b)^2 = a^2 - 2ab + b^2$. Check: $49 - 42 + 9 = 16 = 4^2$.

Model 3: $a^2 - b^2 = (a + b)(a - b)$

1. Cut a 7 by 7 square and remove a 3 by 3 square from one corner. An L-shape is left, with area $a^2 - b^2$.
2. Cut the L into two rectangles: one 4 by 7 and one 4 by 3.
3. Put them end to end. They make one long rectangle 4 cm wide and 10 cm long.

The width is $a - b$ and the length is $a + b$. So $a^2 - b^2 = (a + b)(a - b)$. Check: $49 - 9 = 40 = 10 \times 4$.

Identities as shortcuts

Problem	Identity	Working
103^2	$(a + b)^2$	$100^2 + 2 \times 100 \times 3 + 3^2 = 10,000 + 600 + 9 = 10,609$
98^2	$(a - b)^2$	$100^2 - 2 \times 100 \times 2 + 2^2 = 10,000 - 400 + 4 = 9,604$
52×48	$(a + b)(a - b)$	$50^2 - 2^2 = 2,500 - 4 = 2,496$

REMEMBER

$(a + b)^2$ is not $a^2 + b^2$. The model makes the mistake obvious: the two yellow rectangles are missing.

DID YOU KNOW

The ancient Greek mathematician Euclid proved results like these using areas of squares and rectangles, much as your card pieces do.

CHAPTER 11

Data, chance and heights

Bar graphs

You need: a survey question, a notebook and empty matchboxes.

1. Ask 30 classmates one question, such as "What is your favourite fruit?", and record the answers with **tally marks**: four upright strokes and a fifth crossing them for each group of five.
2. Count each tally: mango 12, banana 7, guava 6 and apple 5.
3. For a 3D bar graph, glue matchboxes in columns on a board, one matchbox for each student.
4. Label the axes, give the graph a title, and keep the bars the same width with equal gaps between them.

On a paper bar graph, choose a **scale**, such as 1 cm for each student, and write it on the graph.

The probability spinner

Probability is a measure of how likely something is, from 0 (impossible) to 1 (certain).

probability = number of ways it can happen $\div$ total number of equally likely outcomes

You need: a card circle, a protractor, a split pin and a card arrow.

1. Divide the circle into 8 equal sectors of 45° .
2. Colour two sectors red, two blue, two green and two yellow.
3. Fix the arrow at the centre with a split pin, loosely enough to spin freely.

The chance of landing on red is $2/8 = 1/4$.

EXAMPLE

Zoya from Srinagar spun 40 times and got red 12 times. Her **experimental probability** was $12/40 = 0.3$, a little above the expected 0.25. After 200 spins she had 52 reds: $52/200 = 0.26$, much closer. The more trials you do, the closer the result usually gets to the expected value.

The clinometer

A **clinometer** measures the **angle of elevation**: the angle between the level ground and your line of sight up to the top of an object.

You need: a protractor with a hole at the centre of its straight edge, a drinking straw, tape, thread and a small weight such as a washer.

1. Tape the straw along the straight edge of the protractor.
2. Tie the thread through the centre hole and hang the weight from it, so the thread hangs straight down across the scale.
3. Look through the straw at the top of a tree or flagpole. Ask a friend to read where the thread crosses the scale.
4. The angle of elevation is the difference between that reading and 90° .

In a right-angled triangle, the **tangent** (tan) of an angle is the opposite side divided by the side next to it. Here, the opposite side is the height above your eyes, and the side next to the angle is your distance from the object. So:

height = distance $\times$ tan(angle) + your eye height

Angle	30°	40°	45°	50°	60°
tan	0.58	0.84	1.00	1.19	1.73

EXAMPLE

Dev stood 15 m from the school flagpole in Dehradun. The angle of elevation was 40°, and his eyes were 1.4 m above the ground. Height = $15 \times 0.84 + 1.4 = 12.6 + 1.4 = 14.0$ m.

TIP

Walk backwards or forwards until the angle is exactly 45°. Since $\tan 45^\circ = 1$, the height above your eyes equals your distance from the object.

SAFETY

Never look at the sun through the straw or any other tube.

CHECK YOURSELF

Practice questions

1. What is the place value and the face value of 6 in 46,215?
2. Write 4,08,15,230 in words in the Indian system.
3. On a spike abacus, what do you do when a tenth bead is added to a skewer?
4. Use the walking rules to find $-3 - 4$.
5. Use zero-pair counters to find $-2 + 7$.
6. The temperature in Shimla was 3 °C and fell by 7 degrees. What was the new temperature?

- 7.** Is the sum of two odd numbers odd or even? Explain using pairs.
- 8.** Why is 1 neither prime nor composite?
- 9.** In the sieve of Eratosthenes up to 100, why can you stop after crossing out the multiples of 7?
- 10.** Why is the fraction wall made from strips 24 cm long?
- 11.** Use the fraction wall lengths to show that $\frac{1}{4} + \frac{1}{6} = \frac{5}{12}$.
- 12.** What angle does each sector have when a circle is divided into sixths?
- 13.** Describe how tearing the corners of a paper triangle shows that its angles add up to 180° .
- 14.** What do the angles of a hexagon add up to?
- 15.** Why does a regular pentagon not tessellate?
- 16.** A right-angled triangle has sides of 5 cm and 12 cm around the right angle. Find the hypotenuse.
- 17.** How does a mason use the numbers 3, 4 and 5 to check a corner?
- 18.** Rectangles with a perimeter of 20 cm are made from a loop of string. Which one has the largest area, and what is that area?
- 19.** Find the area of a circle of radius 14 cm, taking π as $\frac{22}{7}$.
- 20.** How many cubic centimetres are there in one litre?
- 21.** Find the volume of a cuboid 8 cm long, 5 cm wide and 3 cm high.
- 22.** Use an identity to find 101^2 .
- 23.** Use an identity to find 47×53 .
- 24.** A spinner has 5 equal sectors: 2 red and 3 blue. What is the probability of landing on blue?
- 25.** You stand 20 m from a tree and the angle of elevation of its top is 45° . Your eyes are 1.5 m above the ground. How tall is the tree?

Answers

1. Place value 6,000; face value 6.
2. Four crore eight lakh fifteen thousand two hundred thirty.
3. Take all ten beads off and put one bead on the next skewer to the left. This is carrying.
4. Start at -3, face the negative end and walk forwards 4 steps: -7.
5. Put down 2 red and 7 green, remove 2 zero pairs, and 5 green are left: +5.
6. $3 - 7 = -4$, so the new temperature was -4°C .
7. Even. Each odd number has one cap left over, and the two leftover caps make a pair.
8. A prime has exactly two factors, and 1 has only one factor, itself.
9. The next prime is 11, and $11 \times 11 = 121$ is more than 100. Every composite number up to 100 has a factor of 7 or less, so it is already crossed out.
10. Because 24 divides exactly by 2, 3, 4, 6, 8 and 12, so every piece has a whole-number length.
11. A quarter is 6 cm and a sixth is 4 cm. Together they make 10 cm, which matches five twelfths (5×2 cm). So $\frac{1}{4} + \frac{1}{6} = \frac{5}{12}$.
12. $360^{\circ} \div 6 = 60^{\circ}$.
13. Tear off the three corners and place their points together on a straight line. They fit exactly along it, and a straight line is 180° .
14. A hexagon has 6 sides, so $(6 - 2) \times 180^{\circ} = 720^{\circ}$.
15. Each angle is 108° . Three make 324° , which leaves a gap, and four make 432° , which overlap. No whole number of angles makes 360° .
16. $5^2 + 12^2 = 25 + 144 = 169$, and the square root of 169 is 13, so the hypotenuse is 13 cm.
17. The mason marks 3 units along one wall and 4 along the other. If the marks are exactly 5 units apart, the corner is a right angle.
18. The 5 cm by 5 cm square, with an area of 25 cm^2 .
19. $22/7 \times 14 \times 14 = 616 \text{ cm}^2$.
20. $1,000 \text{ cm}^3$.
21. $8 \times 5 \times 3 = 120 \text{ cm}^3$.
22. $(100 + 1)^2 = 10,000 + 200 + 1 = 10,201$.
23. $(50 - 3)(50 + 3) = 50^2 - 3^2 = 2,500 - 9 = 2,491$.
24. $\frac{3}{5}$.
25. Height = $20 \times 1 + 1.5 = 21.5 \text{ m}$.

Glossary

Angle of elevation

the angle between the level ground and your line of sight up to an object.

Area

the amount of flat space inside a shape, measured in square units.

Circumference

the distance all the way round a circle.

Clinometer

an instrument for measuring the angle of elevation.

Composite number

a whole number with more than two factors.

Denominator

the bottom number of a fraction, showing how many equal parts the whole is divided into.

Equivalent fractions

fractions that have the same value, such as $\frac{1}{2}$ and $\frac{3}{6}$.

Face value

the value of a digit by itself, whatever its place.

Hypotenuse

the longest side of a right-angled triangle, opposite the right angle.

Identity

an equation that is true for every value of its letters.

Integer

any whole number, positive or negative, or zero.

Net

a flat shape that can be folded to make a solid.

Numerator

the top number of a fraction, showing how many parts are taken.

Perimeter

the distance all the way round a shape.

Pi (π)

the circumference of any circle divided by its diameter, about 3.14 or $\frac{22}{7}$.

Place value

the value a digit has because of its place in a number.

Prime number

a whole number with exactly two factors, 1 and itself.

Probability

a measure from 0 to 1 of how likely something is to happen.

Radius

the distance from the centre of a circle to its edge.

Tally marks

strokes used to count, grouped in fives.

Tangent (tan)

in a right-angled triangle, the opposite side divided by the side next to the angle.

Tessellation

a pattern of shapes that covers a surface without gaps or overlaps.

Volume

the amount of space a solid takes up, measured in cubic units.

Zero pair

one positive and one negative counter, which together make zero.



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